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When, as is usually the case, dividends on the bond are paid semiannually and it is valued to net the purchaser a nominal rate j convertible twice a year, formula (12) becomes

$$k = a_{\overline{2n}|}\,(g - j)/2, \qquad \text{at rate } j/2. \tag{13}$$

In the simplest of all cases, dividends payable annually at rate g, interest compounded annually at the effective rate i, and the bond maturing in one sum at the end of n years, (12) reduces to the well-known form

$$k = a_{\overline{n}|}\,(g - i). \tag{14}$$

This formula admits of a simple interpretation because it states that the premium per unit of the sum to be redeemed is equal to the present value of an annuity whose annual rent is equal to the excess of the rate of dividend over the rate of interest desired to be realized by the purchaser. I may add that practically all the formulas in this paper admit of a direct interpretation. The interpretation of the final formula usually suggests a simple method of deriving it by general considerations and always throws a great deal of light upon the nature of the problem. It is not my purpose, however, to enter at this time upon the subject of interpretation of interest formulas.

QUESTIONS AND DISCUSSIONS.

Edited by U. G. Mitchell.

NEW QUESTIONS.

24. The following facts are significant:

(1) The New England Association of Mathematics Teachers has appointed a committee "to investigate the current criticisms of high school mathematics."

(2) A committee of the Council of the American Mathematical Society has under consideration the question "whether any action is desirable on the part of the Society in the matter of the movement against mathematics in the schools."

(3) At the recent meeting in Cincinnati of the National Education Association an iconoclastic discussion on the topic: "Can algebra and geometry be reorganized so as to justify their retention for high school pupils not likely to enter technical schools?" aroused approbation and applause. An outline of the remarks by one of the speakers will be printed in this column next month.

In view of these facts what should be done by those who believe in the value of mathematics as a general high school study?

REPLIES.

9. What is the present state of experience with coördinated courses in high school mathematics? What contribution does this promise to the development of mathematics teaching in high schools? What about the corresponding matters in college mathematics? (*Note.*—An individual correspondent need not answer all the questions in number 9; it is sufficient if he answers only one.)

Reply by Roy Cumins, Columbia University, N. Y.

At present there exists in the United States a decided movement toward breaking down the barriers that have hitherto kept separate the various branches of secondary mathematics. These branches are, let us say, algebra, geometry, trigonometry, and an extension of arithmetic; but this discussion will deal primarily with algebra and geometry.

At least three words are popularly employed to designate this movement. They are fusion, unification, and correlation. Much of the difference of opinion that has been expressed with regard to the movement grows out of the fact that these words are not defined in their relation to the teaching and the subject-matter of secondary mathematics. Moreover, a person who might favor correlation opposes, say, unification, without mentioning correlation, and thus is inferentially antagonistic to all phases of the movement.

Let us inquire, then, concerning the meaning of the three words as applied to secondary mathematics. In the first place they concern not so much the content of the subject-matter of algebra and geometry as they do the arrangement and treatment of this subject-matter. This statement cannot be pushed too far, as the texts bearing the names of Myers, Cobb, Long and Brenke, and Short and Elson touch more intimately the actual out-of-school life of the pupil and find the source of more of their applications in such fields as physics, mechanics, biology, engineering, design, architecture, and the trades and industries than did the texts commonly used a decade ago. But the basis for defining our terms must chiefly be sought elsewhere.

Of the three words, "fusion" is certainly the strongest. It may mean that the subject-matter of algebra and that of geometry are to be "fused" in the way in which metals are fused to form an alloy or amalgam. Or, if we do not go quite so far as that, it means fusion to the extent to which the theorems of plane geometry are fused in the ordinary high-school text in geometry. Finally, the word may signify only the fusion which is exhibited in the ordinary high-school text in algebra. Certainly it should mean at least this much.

At the other end of the list, "correlation" may mean almost anything. No doubt we have always been correlating the subject-matter of algebra and geometry to some extent. It would be very difficult and unusual, although not impossible, to teach algebra for a year or a year and a half and then teach geometry for a year without employing in some measure the symbolism, facts, and tools of algebra.

Of course, what the advocates of the present movement wish is close and frequent correlation between the sciences of function and form. They wish coöperation and the penetration of the water-tight (may we say thought-tight?) barrier that we must admit has too effectually kept apart algebra and geometry.

Between the two terms spoken of comes "unity." To me it implies less than "fusion" and more than "correlation." As used in theme writing in my school days, it meant that everything in the subject-matter should bear on the central thought, the purpose, the organizing principle. Nothing irrelevant is to be admitted. To determine what should be admitted into a unified course in secondary mathematics, it would therefore be necessary to decide first upon an organizing principle for secondary mathematics. That task remains to be done.

Many advocates of the plan to relate more closely algebra and geometry continually cite what other countries, especially Germany and France, have done in this regard. For what is said here concerning the teaching of mathematics in these countries I am indebted to the writings of Professors Farrington, Myers, and J. W. A. Young, and to instruction by Professor C. B. Upton.

In France, says Professor Farrington, "Throughout the mathematics courses one is impressed with the intimate relations existing among the various subjects. Arithmetic is not carried to a certain point, there to give way to algebra, in its turn, perhaps, to be supplemented by geometry, but from the fifth form in one division and from the fourth form in another, at least two subjects are run conjointly." So what exists in France is not fusion or unification but parallelism of treatment and correlation. From the time algebra, geometry, and science are begun they are carried along side by side to the end of the course.

Professor Young summarizes the work in Prussia as follows: "In the German schools the subject of study is *mathematics*, and its various branches are studied side by side. At no time is only one subject being studied with the exception of the two lowest years, in which only arithmetic is taken up."

Not long ago, Professor Klein, in a conversation with Professor Upton, expressed his amusement at a seemingly prevalent idea in America that the Germans no longer teach algebra and geometry as such but fuse the two into a course in mathematics. What he says they do is to teach each of the subjects as such and let them help each other whenever they can. Clearly here again is parallelism of treatment with considerable correlation.

Professor Myers says, "The dominant phase of European secondary mathematics is of the mixed, or parallel, or quick alternation type." This does not sound like fusion or unification, as I have defined them.

What do American texts do in "fusing" algebra and geometry? For the most part they do not fuse the two at all, not even the newest texts. Their common plan is to give a chapter or two on algebra, then devote the next pages to geometry, with little or no reference to the algebra that preceded; and then follow with another chapter on algebra that is almost entirely unrelated to the geometry that preceded. (I have not considered the Myers texts in this regard, as they are now being revised, nor the Cobb text, as it is not to my knowledge extensively used in high schools.)

Why should we give parallel courses in geometry and algebra instead of the traditional tandem courses? As it is, one class contends that algebra should precede, while another asks that geometry be taught first. To a high school freshman the beginnings of geometry are certainly more simple, concrete, and familiar than those of algebra. They relate to things that the pupil actually sees out in the world or can construct for himself. On the other hand, the easier parts of algebra are far simpler than the more difficult parts of geometry. If the two were taught simultaneously, each could be of great assistance to the other and correlation could be much more easily effected than is now done. In the third place, the pupil would not rush at high pressure over each subject as he now does. A boy in a German school, for example, studies algebra for six years and geometry for eight years. Time for the maturing of thought and for the effects of retroactive inhibition are thus afforded. Pedagogically, then, we should have parallelism of treatment.

Furthermore, as Professor Young has pointed out, we are not compelled to conduct an experiment to demonstrate the practicability of our theory. France and Germany have done that for us and they tell us with assurance that a great gain has been accomplished. What I favor, therefore, is that both algebra and geometry be begun early, with geometry first, that the study of each be extended over a far greater time than is done at present, and that we do our best in bringing the subject-matter, symbolism, and methods of each subject to assist the other. This should be called parallelism with all possible correlation.

19. How many known proofs are there of the proposition that the square of the hypotenuse of a right-angled triangle is equal to the sum of the squares of the other two sides? Where are the proofs to be found?

REPLY BY C. E. HORNE, Westminster College, Colorado.

As a possible reply to the above question I should like to submit the following:

JURY WIPPER, *Sechsundvierzig Beweise des Pythagoräischen Lehrsatzes.* Aus dem Russischen von F. Graap, Leipzig, 1880.

If there is anything more complete than this work, I shall be glad to learn of it.

(*Note.*—It may be worth while to mention also, in this connection, the earlier and less complete work of IGNAZ HOFFMAN, *Der Pythagoräischen Lehrsatz mit 32 Beweisen,* Mainz, 1819.—EDITOR.)

REPLY BY H. E. SLAUGHT, The University of Chicago.

It may be of interest that the first class in geometry which I taught, in 1883, became "Pythagorean crazy," and that we elaborated over forty proofs, each being carefully copied on stiff cardboard and all bound together and deposited in the archives of the school, Peddie Institute, Heightstown, N. J. I hope they are still preserved.

22. What can the colleges do toward improving the teaching of mathematics in secondary schools?

REPLY BY CHARLES N. MOORE, University of Cincinnati.

It is evidently desirable that colleges and universities should do everything they reasonably can to improve the teaching of preparatory subjects in the secondary and primary schools in their localities. Not only is this their duty from the broad viewpoint of the relation of such institutions to society; but, from the narrower viewpoint of benefits to the institutions themselves, any results achieved in this direction will react in a stimulating way upon the college or university. Consequently it seems worth while to give here a brief account of what the department of mathematics at the University of Cincinnati is and has been doing in that direction for the past fifteen years.

The University of Cincinnati, being a municipal institution, has always made a special effort to coöperate as much as possible with all municipal activities, and particularly with all activities in other subdivisions of the public educational system of the city. In line with this general policy, courses especially designed for those who are actively engaged in teaching have been given by all the departments in the late afternoon hours and on Saturdays. These courses have been highly appreciated and well attended by teachers in the city and in the neighboring towns.

In the department of mathematics at least one such course, and sometimes three or more, have been given each year during the past fifteen years. These courses have ranged from freshman college courses to advanced graduate courses, in order to meet the needs and interests of as many teachers in the vicinity as possible. Very few of them were definitely method courses, for it is a conviction of the department, based on its own experiences, that most teachers of primary or secondary subjects who have had some experience in actual teaching will profit more from additional information in their subject than they will from specific training in methods. Of course incidental discussion of pedagogical questions is valuable, and we have always kept that in mind. But our principal effort has been to give the teachers in the preparatory schools of our vicinity a wider mathematical horizon.

As a result of our efforts in this direction almost all the younger teachers of mathematics in the Cincinnati high schools have taken one or more courses in mathematics at the University, and quite a number of them have taken the master's degree in mathematics. Some of these latter did all their mathematical study from freshman college work through to the master's degree in connection with these courses given especially for teachers. Also quite a number of teachers in the primary schools, including several who are now principals of schools in the city, and a number of teachers in preparatory schools of neighboring towns, have taken one or more courses in mathematics.

Another feature of the work done at the University of Cincinnati to bring about closer coöperation with the teachers in the preparatory schools is an annual gathering at the University of teachers in the accredited secondary schools. Those in attendance are grouped into sections according to the subjects in which their chief interest lies, and pedagogical questions relating to secondary schools are discussed. The members of the different departments of the university make it a point to attend the sectional meetings in which their subjects are discussed, and not only participate in the discussion but frequently give one of the papers of the day. The papers presented in the mathematical section and the ensuing discussions have been found to be extremely interesting and helpful to all those attending, and there is no doubt that these annual meetings have done much to increase the feeling of a community of interest between teachers in the university and those in the secondary field.

The chief apparent effect, as noticed by the department of mathematics, of this steady effort to do something toward the further training of teachers in the preparatory schools has been to unify the teaching of mathematics in this vicinity and to bring about a closer coöperation among all those engaged in such teaching, whether the subject taught be arithmetic or the theory of functions. We are thoroughly convinced that the value of this unification and coöperation makes the effort put forth to obtain it seem very slight in comparison. We are also convinced that there are other beneficial effects of this effort which are less apparent but not less important. And, finally, we believe that any college or university that has done little or no work of this sort will find it highly worth while to increase its activity in this direction.

BOOK REVIEWS.

EDITED BY W. H. BUSSEY, University of Minnesota.

Analytic Geometry. By L. WAYLAND DOWLING and F. E. TURNEAURE. Henry Holt and Co., New York, 1914. xi + 266 pages.

A chief feature of several recent texts on analytics is the emphasis of the general idea of a function and its graph rather than of the theory of conics. This text retains that emphasis to a large extent. After 25 pages devoted to chapters on "Systems of Coördinates" (I) and "Directed Segments and Areas of Plane Figures" (II), we find 65 pages of discussion of "Functions and their Graphic Representation" (III), "Loci and their Equations" (IV), and "Equations and their Loci" (V), the last chapter including "Transformation of Coördinates." In Chapter III first methods of graphing functions are given with illustrations from algebraic and transcendental functions in rectangular and polar coördinates. The equation of a locus is defined in Chapter IV and the standard equations of straight lines, the conics and Cassinian ovals are derived for both coördinate systems. Chapter V gives methods of discussing an equation with numerous examples. It is not until Chapter VI, "Loci of First Order," beginning on page 98, that we find a systematic treatment of the straight line. After what has preceded, 10 pages suffices. Then in Chapter VII, "Loci of Second Order, Equations in Standard Form," we find, compassed in 34 pages, a fairly complete elementary treatment of the conics, including "Poles and Polars" and "Systems of Conics." The next chapter treats the general equation of second degree.

In accord with another modern development, we find (included in Chapter IX, "Loci of Higher Order and Other Loci") a twelve page discussion of "Empirical Equations and their Loci." This subject, on account of its importance in applications of mathematics to the sciences, seems destined to become an essential part of a good course in analytics. The presentation of the authors, which includes the use of logarithmic coördinate paper, is excellent.

Following these chapters on plane analytics is a brief treatment (about 50 pages) of solid analytics. There is little of novelty in this part of the book.

The book as a whole impresses one very favorably. The general order of presentation is excellently adapted to give the student a real appreciation of the power and beauty of analytic geometry, and also the ability to use it. The tendency to lay a little more stress than usual on the geometrical aspect of the